ECS 332: Principles of Communications **2016/1**

HW 5 — Not Due

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Problem 1. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 5.1. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

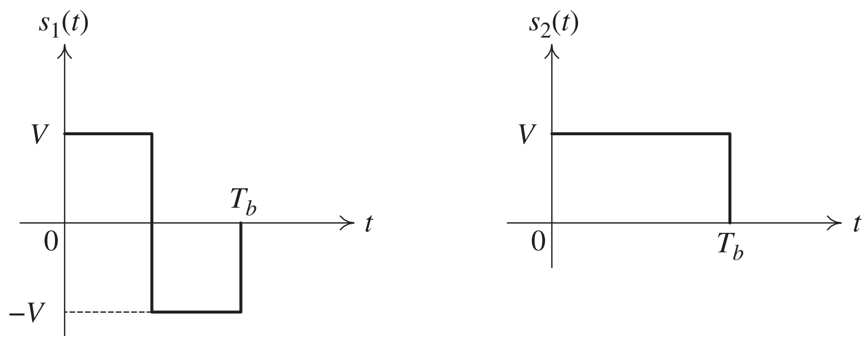


Figure 5.1: Signal set for Question 1

(a) Find the energy in each signal.

$$E_{s_1} = \int_{-\infty}^{\infty} |s_1(t)|^2 dt = \int_0^{T_b} V^2 dt = V^2 T_b$$

$$E_{s_2} = \int_{-\infty}^{\infty} |s_2(t)|^2 dt \quad ''$$

(b) Are they energy signals?

V and T_b are some positive constants $\Rightarrow 0 < V^2 T_b < \infty$

$\Rightarrow$ Energy signal. Yes.

(c) Are they power signals?

No. Energy signal can not be a power signal.

(d) Find the (average) power in each signal.

Any energy signal has 0 average power.

(e) Are the two signals $s_1(t)$ and $s_2(t)$ orthogonal?

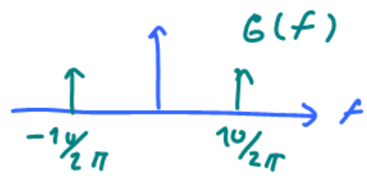
$$\langle s_1(t), s_2(t) \rangle \equiv \int_{-\infty}^{\infty} s_1(t)\, s_2^*(t)\, dt = 0 \Rightarrow \text{Yes.}$$

Problem 2. (Power Calculation) For each of the following signals $g(t)$, find (i) its corresponding power $P_g = \langle |g(t)|^2 \rangle$, (ii) the power $P_x = \langle |x(t)|^2 \rangle$ of $x(t) = g(t)\cos(10t)$, and (iii) the power $P_y = \langle |y(t)|^2 \rangle$ of $y(t) = g(t)\cos(50t)$

(a) $g(t) = 3\cos(10t + 30°)$.

$$P_g = \frac{3^2}{2} = 4.5$$

$$x(t) = g(t)\cos(10t)$$



(b) $g(t) = 3\cos(10t + 30^\circ) + 4\cos(10t + 120^\circ)$. (Hint: First, use phasor form to combine the two components into one sinusoid.)

(c) $g(t) = 3\cos(10t) + 3\cos(10t + 120^\circ) + 3\cos(10t + 240^\circ)$

Problem 3. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the **energy spectral density** of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. Furthermore, the energy contained in the frequency band I can be found from the integral $\int_I |G(f)|^2 df$ where the integration is over the frequencies in band I. In particular, if the band is simply an interval of frequency from f_1 to f_2, then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \tag{5.1}$$

In this problem, assume

$$g(t) = 1[-1 \leq t \leq 1].$$

(a) Find the (total) energy of $g(t)$.

(b) Figure 5.2 define the main lobe of a sinc pulse. It is well-known that the main lobe of the sinc function contains about 90% of its total energy. Check this fact by first computing the energy contained in the frequency band occupied by the main lobe and then compare with your answer from part (a).

Hint: Find the zeros of the main lope. This give f_1 and f_2. Now, we can apply (5.1). `MATLAB` or similar tools can then be used to numerically evaluate the integral.

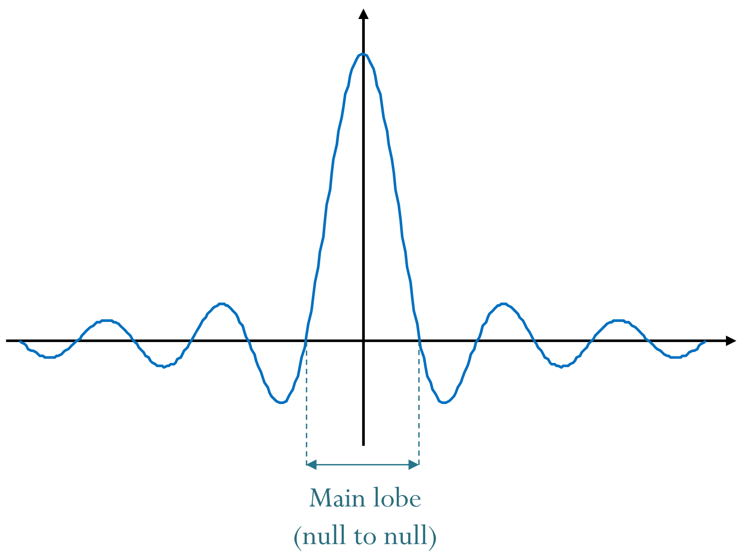


Figure 5.2: Main lobe of a sinc pulse

(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $I = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least 99% of the total energy in $g(t)$.

Problem 4. Consider a "square" wave (a train of rectangular pulses) shown in Figure 5.3. Its values periodically alternates between two values A and 0 with period T_0. At $t = 0$, its value is A.

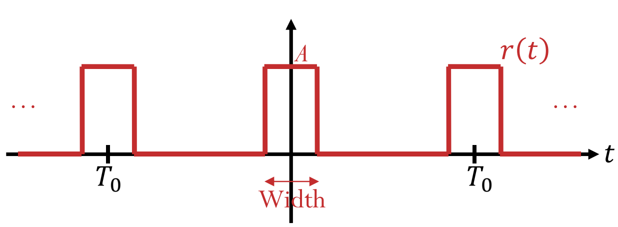


Figure 5.3: A train of rectangular pulses

Some values of its Fourier series coefficients are provided in the table below:

k	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
c_k	$-\frac{\sqrt{2}}{7\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{5\pi}$	0	$\frac{\sqrt{2}}{3\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{3\pi}$	0	$-\frac{\sqrt{2}}{5\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{7\pi}$

(a) Find its duty cycle.

(b) Find the value of A. (Hint: Use c_0.)

Extra Question

Here is an optional question for those who want more practice.

Problem 5 (M2011)**.** In this question, you are provided with a partial proof of an important result in the study of Fourier transform. Your task is to figure out the quantities/expressions inside the boxes labeled a,b,c, and d.

We start with a function $g(t)$. Then, we define $x(t) = \sum_{\ell=-\infty}^{\infty} g(t - \ell T)$. It is a sum that involves $g(t)$. What you will see next is our attempt to find another expression for $x(t)$ in terms of a sum that involves $G(f)$.

To do this, we first write $x(t)$ as $x(t) = g(t) * \sum_{\ell=-\infty}^{\infty} \delta(t - \ell T)$. Then, by the convolution-in-time property, we know that $X(f)$ is given by

$$X(f) = G(f) \times \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{\quad b \quad}\right)$$

We can get $x(t)$ back from $X(f)$ by the inverse Fourier transform formula: $x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$. After plugging in the expression for $X(f)$ from above, we get

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} e^{j2\pi ft} G(f) \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{\quad b \quad}\right) df \\ &= \boxed{\quad a \quad} \int_{-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} e^{j2\pi ft} G(f) \delta\left(f + \boxed{\quad b \quad}\right) df. \end{aligned}$$

By interchanging the order of summation and integration, we have

$$x(t) = \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi ft} G(f) \delta\left(f + \boxed{\quad b \quad}\right) df.$$

We can now evaluate the integral via the sifting property of the delta function and get

$$x(t) = \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} e^{\boxed{\quad c \quad}} G\left(\boxed{\quad d \quad}\right).$$